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Three-Dimensional Rainbow Schlieren Measurements in Underexpanded Sonic Jets from Axisymmetric Convergent Nozzles

Hirofumi TAKANO[1], Daiyu KAMIKIHARA[1], Daisuke ONO[2], Shinichiro NAKAO[2], Hideki YAMAMOTO[3], and Yoshiaki MIYAZATO[4]

1. Graduate Student, The University of Kitakyushu, 1-1, Wakamatsu, Kitakyushu, Fukuoka, 808-0135, Japan
2. Associate Professor, The University of Kitakyushu, 1-1, Wakamatsu, Kitakyushu, Fukuoka, 808-0135, Japan
3. Engineering Adviser, The University of Kitakyushu, 1-1, Wakamatsu, Kitakyushu, Fukuoka, 808-0135, Japan
4. Professor, The University of Kitakyushu, 1-1, Wakamatsu, Kitakyushu, Fukuoka, 808-0135, Japan

The rainbow schlieren deflectometry has been combined with the computed tomography to obtain three-dimensional density fields of shock containing free jets and we call the method the schlieren CT. Experiments on the schlieren CT have been performed at a nozzle pressure ratio of 4.0 by using an axisymmetric convergent nozzle with an inner diameter of 10 mm at the exit where the nozzle was operated at an underexpanded condition. Multi-directional rainbow schlieren pictures of an underexpanded sonic jet can be acquired by rotating the nozzle about its longitudinal axis in equal angular intervals and the three-dimensional density fields are reconstructed by the schlieren CT. The validity of the schlieren CT is verified by a comparison with the density fields reconstructed by the Abel inversion method. As a result, it is found that excellent quantitative agreement is reached between the three-dimensional jet density fields reconstructed from both methods.

Keywords: Rainbow schlieren, Computer tomography, Convergent nozzle, Sonic jet

Introduction

To understand the prominent characteristics of complex shock-containing jet flows, schlieren methods [1] are widely used as optical tools because of its simple optical arrangement with a high degree of resolution and ability to easily observe such structures as jet shear layers, a Mach disk, barrel shock waves, slip streams, Prandtl-Meyer compression and expansion waves in supersonic flows. Also, this method for flow visualization does not require the introduction of additives into the flowfield and is capable of providing useful qualitative information on the variations in fluid density, temperature, and static pressure. The conventional schlieren techniques have been extensively employed for qualitative flow visualization.

On the other hand, the simplest method for visualizing jet flow fields with varying refractive index quantitatively is probably the rainbow schlieren deflectometry [2, 3] which has been widely developed by Agrawal and coworkers. Kolhe and Agrawal [4] measured the flow characteristics of the shock-containing jet issued from a thick orifice plate with sharp upstream edges by the rainbow schlieren deflectometry. Their study was probably the first application of the rainbow schlieren deflectometry for shock containing jets. Yamamoto et al. [5] applied the rainbow schlieren deflectometry for correctly expanded supersonic jets from an axisymmetric conver-

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gent-divergent nozzle with a design Mach number of 1.6. They showed that the density data from the rainbow schlieren for a shock-free jet are in good quantitative agreement with those obtained from their numerical simulation and Pitot tube measurements.

Agrawal et al. [6] applied the rainbow schlieren deflectometry with tomography for a three-dimensional temperature field in a low speed gas jet. Their study demonstrates that the rainbow schlieren tomography can be used for quantitative measurements of density in axisymmetric flows. However, there has been no practical application of rainbow schlieren deflectometry for a quantitative three-dimensional density field measurement in a shock-containing jet. Therefore, the aim of the present study is to develop a three-dimensional rainbow schlieren system for shock-containing jets. For the purpose, the conventional rainbow schlieren deflectometry is combined with the computed tomography and we call the method the schlieren CT in the present paper. The density field of the underexpanded sonic jet from an axisymmetric convergent nozzle has been measured by the schlieren CT. The density field for the same jet flow is also displayed from the so-called Abel inversion method which is extensively used for the reconstruction of the radial density distribution of a cylindrically symmetric object. Since this method is very effective for quantitative density measurements in axisymmetric shock containing jets [4,5], the density field reconstructed from the Abel inversion method is used to confirm that reconstructed from the schlieren CT.

Experimental appratus

Experiments have been performed in a blow-down supersonic wind tunnel with the jet issued in the quiescent laboratory air. A schematic diagram of the experimental apparatus with a schlieren optical system is shown in Fig. 1. The air supplied by a compressor that pressurizes the ambient air up to 1 MPa is filtered, dried and stored in a high-pressure reservoir consisting of two tanks with a total capacity of 2 m^3. The high-pressure dry air from the reservoir is stagnated in a plenum chamber and then discharged into the atmosphere through a test nozzle. The total temperature in the plenum chamber is equal to the room temperature, and the plenum pressure is controlled and maintained constant during the testing by a solenoid valve. The test nozzle used in the present experiment is shown in Fig. 2. It has an axisymmetric wall contour with a 10 mm inner diameter at the exit plane and the nozzle wall contour over a range of parts A to B is designed by a sinusoidal curve to provide uniform parallel flow with respect to the nozzle axis at each location of the A and B. The jet issued from the nozzle is visualized by the rainbow schlieren deflectometry with a field of view of 100 mm diameter for a nozzle pressure ratio of 4.0, i.e., jet flow in underexpanded condition for the present nozzle.

The rainbow schlieren system consists of rail-mounted optical components including a 50 μm diameter pinhole, two 100 mm diameter, 500 mm focal length achromatic lenses, a computer generated 35 mm wide slide with color gradation in a 1.4 mm wide strip, and a digital camera with variable focal length lens. A continuous 250 W metal halide light source connected to a 50 μm diameter fiber optic cable provides the light input at the pinhole through a 16.56 mm focal length objective lens. The camera output in the RGB format is digitized by a personal computer with 24 bit color frame grabber. The rainbow filter

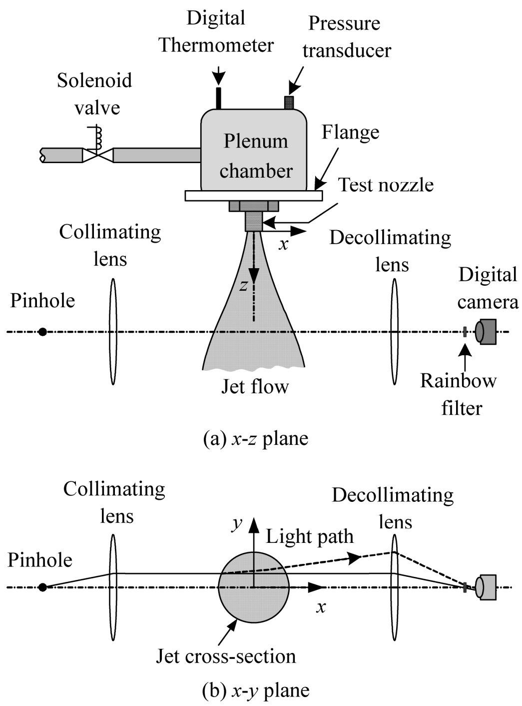
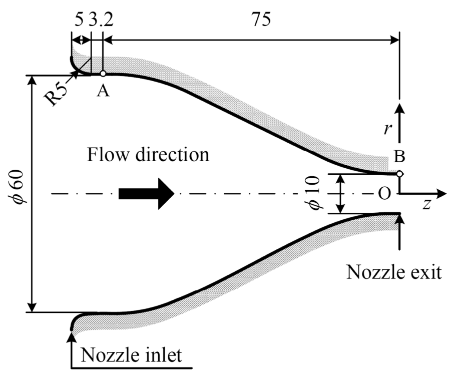


Fig. 1 Schematic drawing of experimental apparatus



Part A to B: $r = 12.5\sin\left[\frac{\pi}{75}(z-37.5)\right] + 17.5$

Fig. 2 Schematic drawing of test nozzle

used in the present experiments is shown in Fig. 3. The filter was fabricated in computer software and then printed digitally on a high resolution 23 mm color film recorder. It has continuous hue variation from *hue* = 0 to 314 deg in a 1.4 mm wide strip and the origin y = 0 corresponds to *Hue* = 180 deg. The characteristics of the rainbow filter are performed by traversing the filter automatically in intervals of 0.1 mm in the y direction at the schlieren cut-off plane before starting experiments. The calibration result is shown as open symbols in Fig. 4. The abscissa is *Hue* and the ordinate is the transverse ray displacement d from the x axis at the cut-off plane. The solid line indicates a least-squares regression line of the experimental data using fifth degree polynomials.

As shown in Fig. 5, the test nozzle is installed inside a pulley with a gear ratio of 1:4 and it can be rotated about the center axis of the nozzle by a stepping motor with a gear ratio of 1:36 connected through a timing belt. In the present experiment, rainbow schlieren pictures are acquired over a range of nozzle angular angles from θ = 0 deg to -180 deg by rotating the nozzle about its longitudinal axis (z axis) in equal angular intervals of -10 deg. These 19 schlieren pictures are used for reconstruction of the jet density field by the Abel inversion and schlieren CT methods. The jet density profiles obtained from both methods are compared with each other.

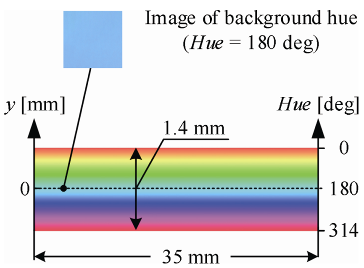


Fig. 3 Rainbow filter image

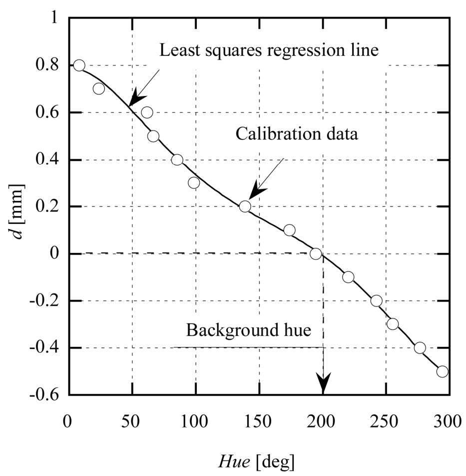


Fig. 4 Calibration curve of rainbow filter

Results and Discussion

Rainbow schlieren visualization

A typical rainbow schlieren picture of underexpanded sonic jet issued from an axisymmetric convergent nozzle is shown in Fig. 6 with the flow from left to right. The ratio p_{os}/p_b of the plenum pressure p_{os} to back pressure p_b is held constant at 4.0. As shown on the upper right corner in the picture of Fig. 6, the rainbow filter is placed horizontal with respect to the z axis at the schlieren cut-off plane and the background hue corresponds to the color of the location indicated as the right pointing arrow for the filter image. The schlieren picture has a high resolution of 55 pixels/mm and it corresponds to a spatial resolution of around 20 μm for the present schlieren set up. The schlieren picture of Fig. 6 shows the archetypal structure of underexpanded jet including the expansion fans, barrel shock, Mach disk, reflected shock, and slip stream. Also, a significant color gradation along the radial direction can be clearly seen near the jet boundaries of y = + 5 mm and - 5 mm. Background hues can be seen at the jet centerline and in the regions radially away from the jet boundaries where the transverse component of the density gradient is intrinsically zero. Also, the hues cor-

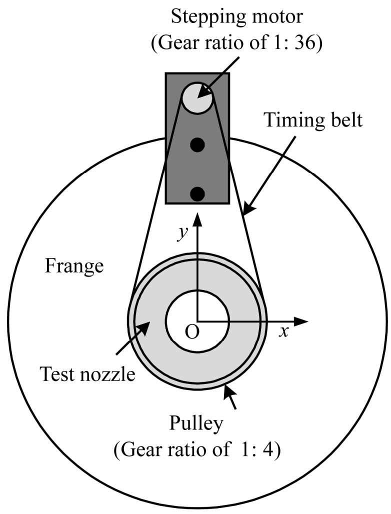


Fig. 5 Nozzle rotating device

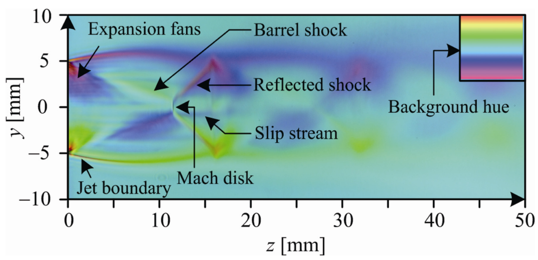


Fig. 6 Typical rainbow schlieren picture of underexpanded sonic jet for p_{os}/p_b=4.0

responding to the upper and lower jet boundaries are different from each other and they exist in the lower and upper regions from the background hue in the filter image shown in Fig. 6, respectively, because an axisymmetric rainbow filter shown in Fig. 3 is used in the present schlieren system. Therefore, the light rays passing through the upper and lower jet boundaries have downward and upward angular deflections toward the opposite boundaries, respectively.

Density contour plots

The density field corresponding to the schlieren picture of Fig. 6 can be estimated from two reconstruction processes. One is the analytical method based upon the Abel inversion and it is extremely effective for the reconstruction of the radial density distribution of a cylindrically symmetric jet. The other is that based upon the schlieren CT and it is valid even for that of an axisymmetric jet. The detail descriptions for the reconstruction of density fields based upon the Abel inversion method and the rainbow schlieren combined with the computed tomography are reported in the paper of Kolhe and Agrawal [4] and that of Agrawal et al. [6], respectively.

The density contour plots reconstructed from both methods are compared with each other as shown in Figs. 7(a) and 7(b) where the density contour plot of Fig. 7(a) is computed as the mean of the three density fields estimated from the Abel inversion using the horizontal schlieren images recorded at nozzle rotational angles of θ = 0, -60 and -120 deg, because, the reconstruction process based upon the Abel inversion method is relatively sensitive to noise and errors in the determination of the jet centerline [3]. Also, Fig. 7 is presented with the color representation of density field and the solid lines with a numeral in the contour show isopycnic.

These contour plots indicate good qualitative and quantitative indications of the shape of the various features of the shock containing jet structure. The density contour of Fig. 7(b) exhibits nearly axial symmetry about the jet centerline. From a comparison between Figs. 7(a) and 7(b), it is found that the global shapes of each shock in the shock cell structure are almost identical to each other. In addition, the axial distances of the Mach disk from the nozzle exit plane are nearly the same for both density fields.

Density distributions along axial and radial directions

A comparison of density profiles along the jet centerline obtained by the Abel inversion and schlieren CT for an underexpanded sonic jet is given in Fig. 8 where density data near the nozzle exit are excluded from Fig. 8, because in areas very close to the nozzle exit, reflections of the schlieren light rays by the exit wall surface lead to larger errors in the density field or yield regions where density data cannot be precisely obtained. The abscissa of Fig. 8 is the downstream distance z from the nozzle exit plane and the ordinate shows the density. Also, the blue and red lines in Fig. 8 indicate the densities reconstructed by the Abel inversion and schlieren CT methods, respectively, and the dashed line parallel to the abscissa is that in ambient air.

For a nozzle pressure ratio of p_{os}/p_b = 4.0 in the present experiment, the density ρ_e at the nozzle exit plane calculated based upon the assumption of the one-dimensional isentropic flow from the nozzle inlet to the exit becomes a value of 3.04 kg/m^3 and it is shown as the left-pointing arrow on the vertical axis in Fig. 8. Therefore, the densities inside the nozzle are always higher than the density ρ_b = 1.20 kg/m^3 in surrounding air. Since the static pressure at the nozzle exit plane is greater than

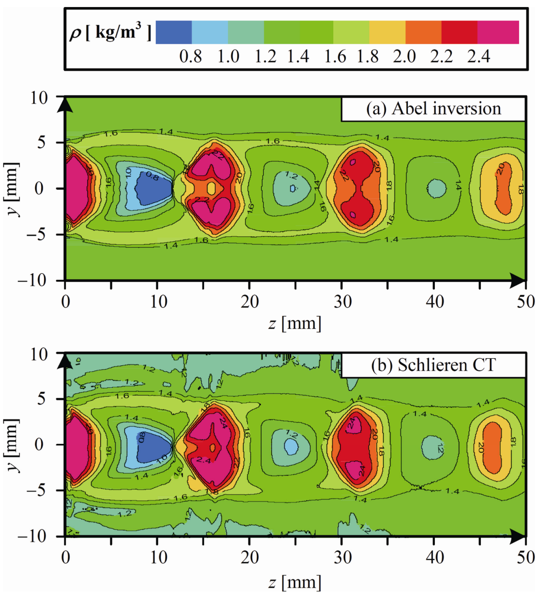


Fig. 7   Axial density contour plots for $p_{os}/p_b = 4.0$

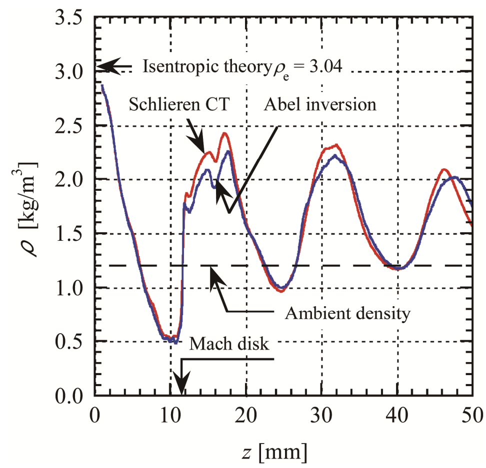


Fig. 8   Axial density profiles for $x = y = 0$ mm

the back pressure, the freestream Mach numbers at the nozzle exit plane are almost unity and Prandtl-Meyer expansion fans occur from the lip of the nozzle exit. A conventional shock cell structure appearing repeatedly in the jet flowfield can be seen in Fig. 8. Also, the shock amplitudes in the shock-cell structure decrease gradually toward the downstream direction and there is very favorable overall agreement between both density profiles over the full length of the region measured. Both the shock-cell intervals and density amplitudes agree extremely well.

In order to observe the three dimensional structure of a shock-containing jet, the cross-sectional density contour plot at any axial location in the jet flowfield was obtained from the schlieren CT using the horizontal rainbow schlieren images taken over 19 viewing angles from θ = 0 to -180 deg by rotating the nozzle about the z axis in equal angular intervals of -10 deg. The result for the jet density field at an axial location of z = 10 mm is depicted in Fig. 9 with the color bar showing density ranges. This axial location exists in the region just prior to the Mach disk.

The density contour plot of Fig. 9 exhibits nearly symmetry about the center of the contour. However, the axis of symmetry does not coincide with $x = y = 0$. It appears that the direction of the optical axis of the rainbow schlieren system in the present experiment may have deviated slightly from that of a vector along x axis of Fig. 1. Also, the isopycnic line with ρ = 1.6 kg/m^3 around 4 mm away from the center of the contour shows a ring with some indentations. It is deemed responsible for the presence of shocklets in the flow observed in the study of Fourguette et al. [7]. However, Krothapalli et al. showed that it results from some imperfections of the nozzle inner surface [8]. Such three-dimensional flow features cannot be easily obtained from the line-of-sight approach such as conventional schlieren and shadowgraph techniques.

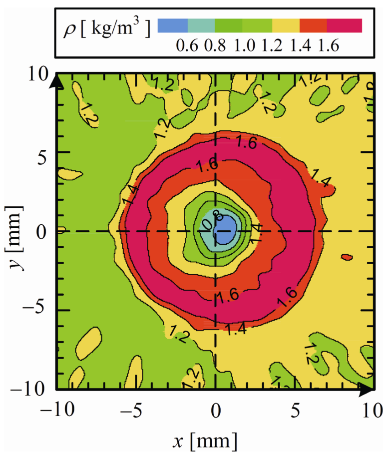


Fig. 9 Cross-sectional density contour plot at z = 10 mm

The radial density profiles reconstructed by the Abel inversion and schlieren CT are shown in Fig. 10 as the blue and red lines, respectively. The profile corresponds to the densities obtained along y axis at x = 0 in Fig. 9. As can be seen in Fig. 10, the jet has lower densities compared with the ambient one over a range of radial distances from r = 0 mm to around 2 mm, which is due to the isentropic expansion from the plenum chamber upstream of the nozzle to just upstream of the Mach disk. Also, from the density profile by the schlieren CT, the density field just upstream of the Mach disk is almost axisymmetric with respect to the horizontal axis including y = 0. Excellent quantitative agreement is reached between the density profiles reconstructed from both analytical methods.

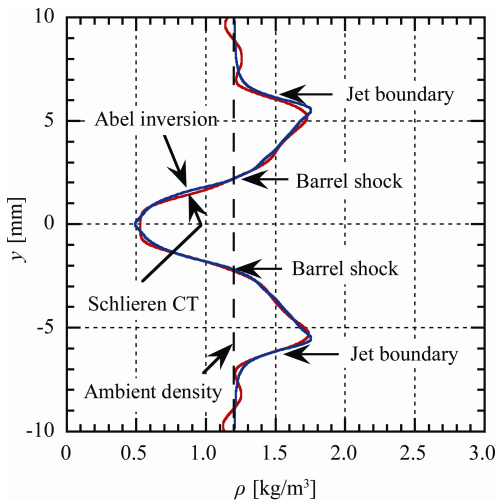


Fig. 10 Radial density profiles at x = 0 mm and z = 10 mm

Concluding Remarks

Three-dimensional density fields of the shock containing free jet issued from an axisymmetric convergent nozzle with an inner diameter of 10 mm at the exit were quantitatively obtained by the rainbow schlieren deflectometry combined with the computed tomography and we call the method the schlieren CT. As a result, a density contour plot at any jet cross-section can be efficiently inferred from the schlieren CT and a density field including the jet axis quantitatively shows the representative shock-cell structure of repeated compression and expansion waves by continuous color gradation. Axial and radial density profiles by the schlieren CT were compared with those by the Abel inversion method. The density profiles obtained from both methods are quantitatively identical to each other.

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